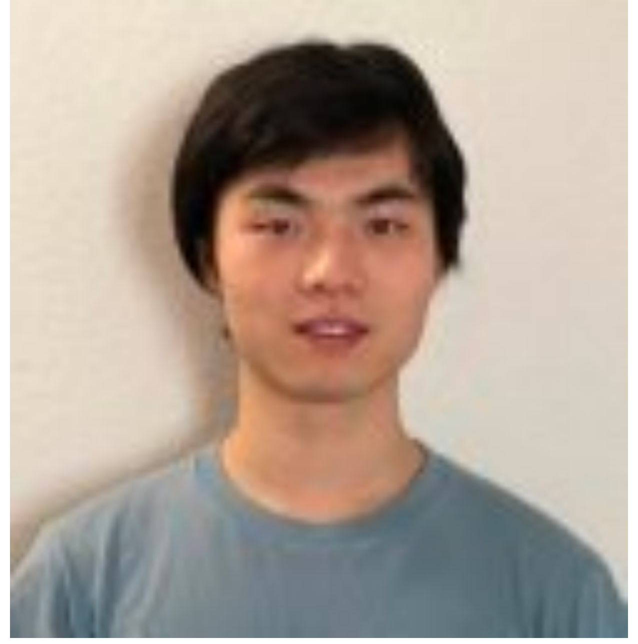
Green’s Function Zeros in Symmetric Mass Generation

Meng Zeng (曾萌)

Max Planck Institute for the Physics of Complex Systems

Acknowledgement

Yi-Zhuang You
UCSD



Da-Chuan Lu
UCSD→
CU Boulder & Harvard

Fu Xu
Nanjing U.

Juven Wang
Harvard

Zheng Zhu
UCAS

NSF National Science Foundation

Outline

- Symmetric mass generation (SMG)
- Emergence of Green's function zeros
- Low energy consequences of zeros?

Outline

- Symmetric mass generation (SMG)
- Emergence of Green's function zeros
- Low energy consequences of zeros?

$G(t,x) \equiv \langle \psi(0,0)\psi^\dagger(t,x)\rangle$

- **Poles**: $G(\mathbf{k},\omega) \to \infty$
  - Free fermion: $G(\mathbf{k},\omega) = \dfrac{1}{\omega - \epsilon_{\mathbf{k}}}$.
  - Interacting: $G(\mathbf{k},\omega) = \dfrac{1}{\omega - \epsilon_{\mathbf{k}} - \Sigma(\mathbf{k},\omega)} = \dfrac{Z_{\mathbf{k}}}{\omega - \tilde{\epsilon}_{\mathbf{k}}}$.
- **Zeros**: $G(\mathbf{k},\omega) \to 0$
  - $\Sigma(\mathbf{k},\omega) \to \infty$.
  - Strongly coupled fixed point.

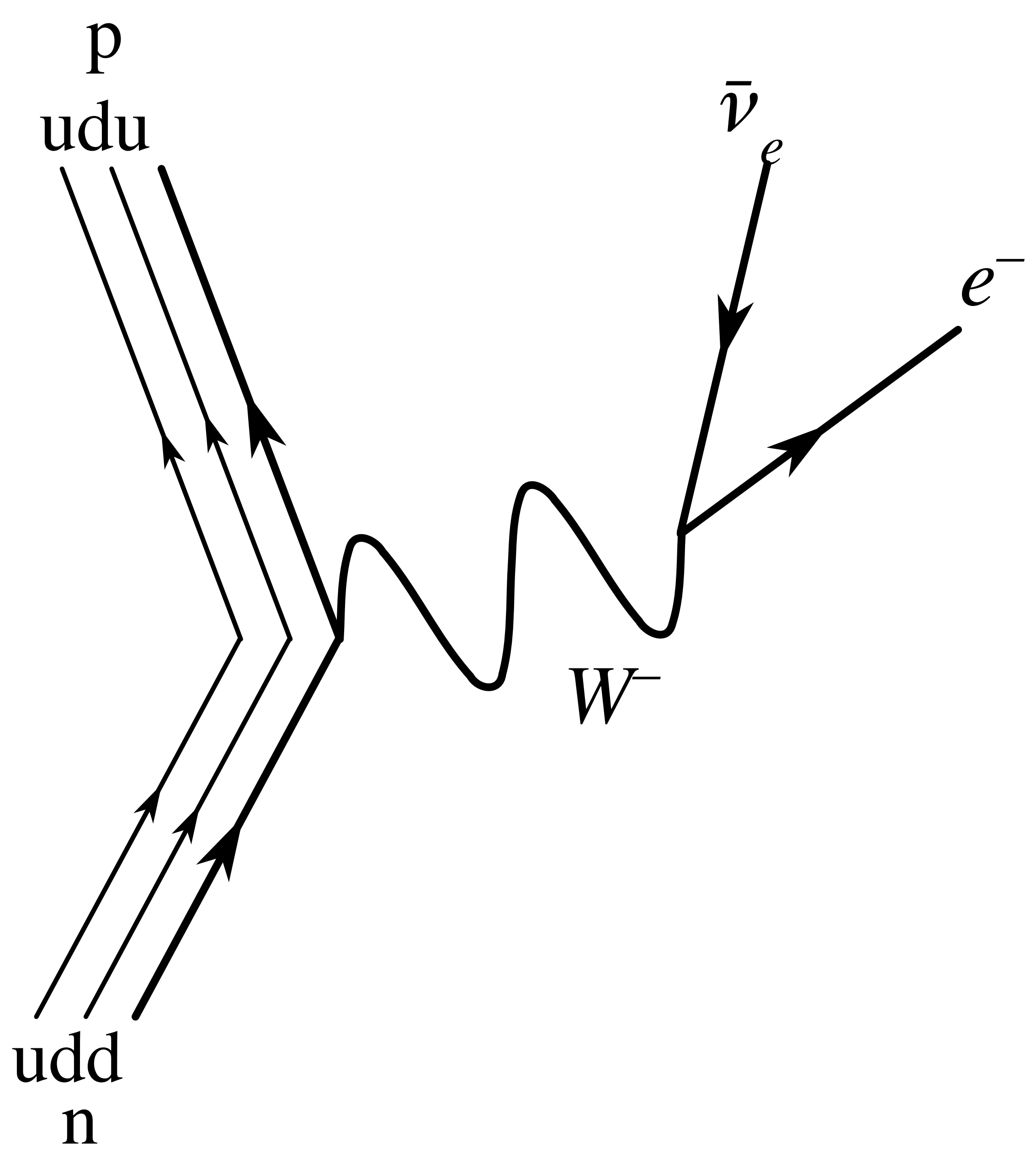
Origin of mass

- Higgs mechanism
    - SSB

Origin of mass

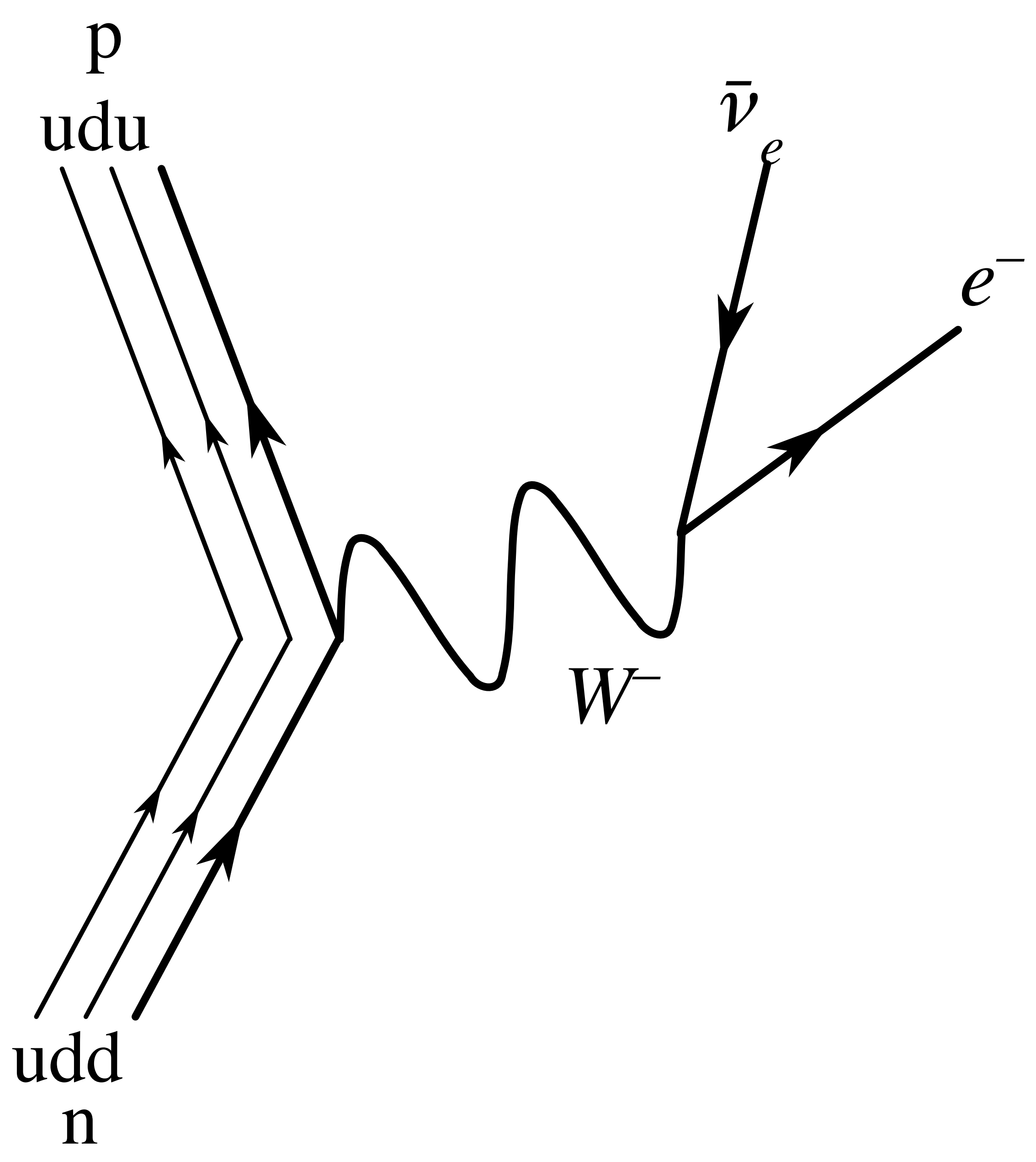
- Higgs mechanism
  - SSB
- Mass gap without SSB
  - Non-trivial ground state: e.g. Mott insulator
  - Trivial ground state: Symmetric mass generation (SMG)

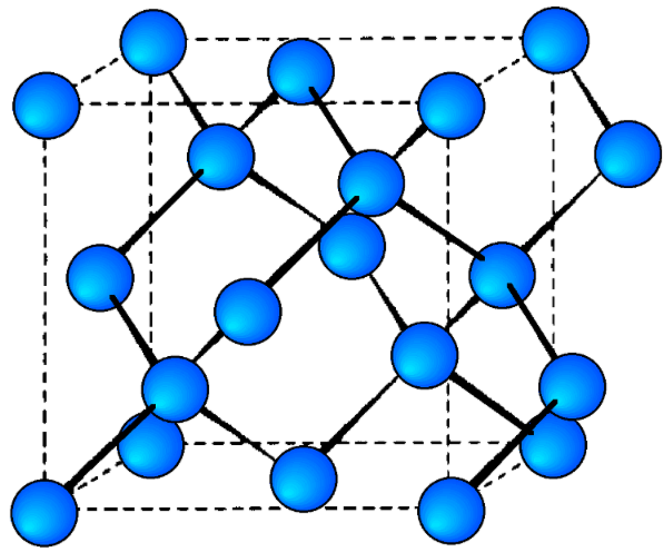
Motivation of studying SMG

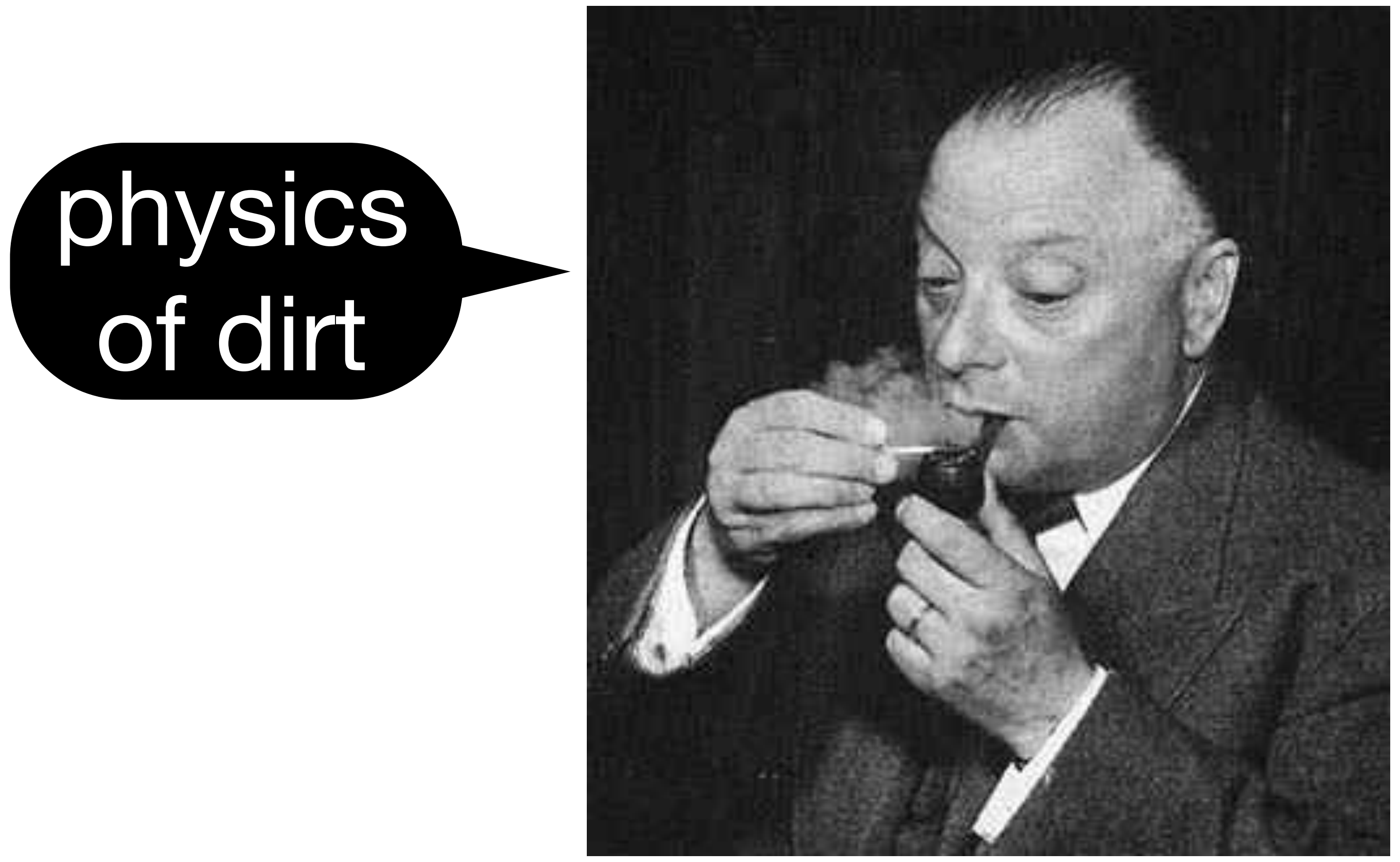


- Chiral lattice gauge theory
  - Mirror fermion (Eichten, Preskill 1986)
  - 3450 chiral fermion (Zeng et al. 2022)
- Strong CP problem in QCD
  - Alternative to Peccei-Quinn (Wang 2022)

Motivation of studying SMG



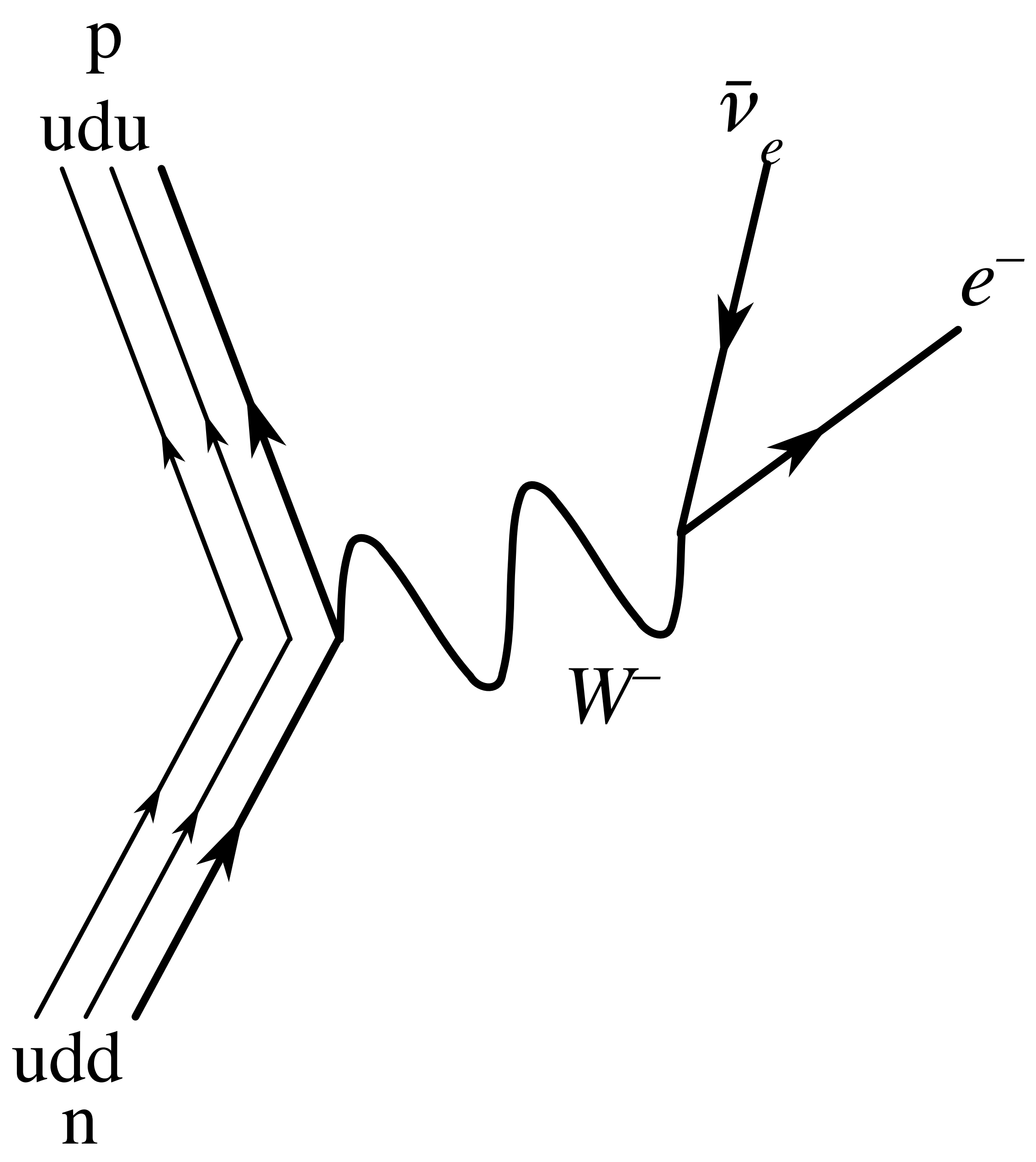


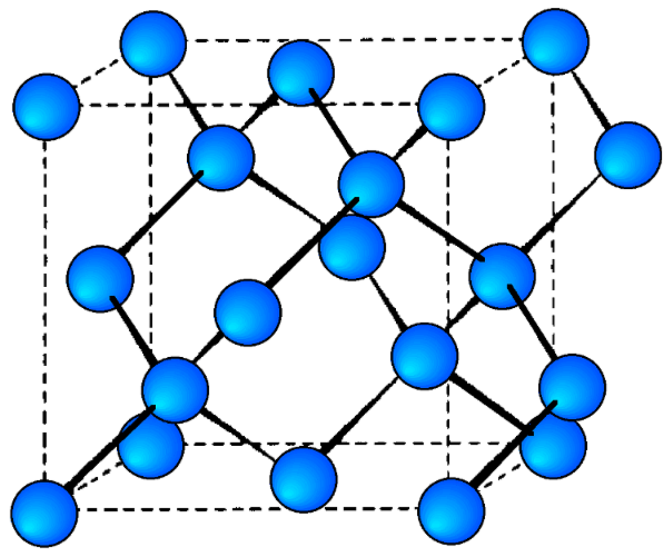


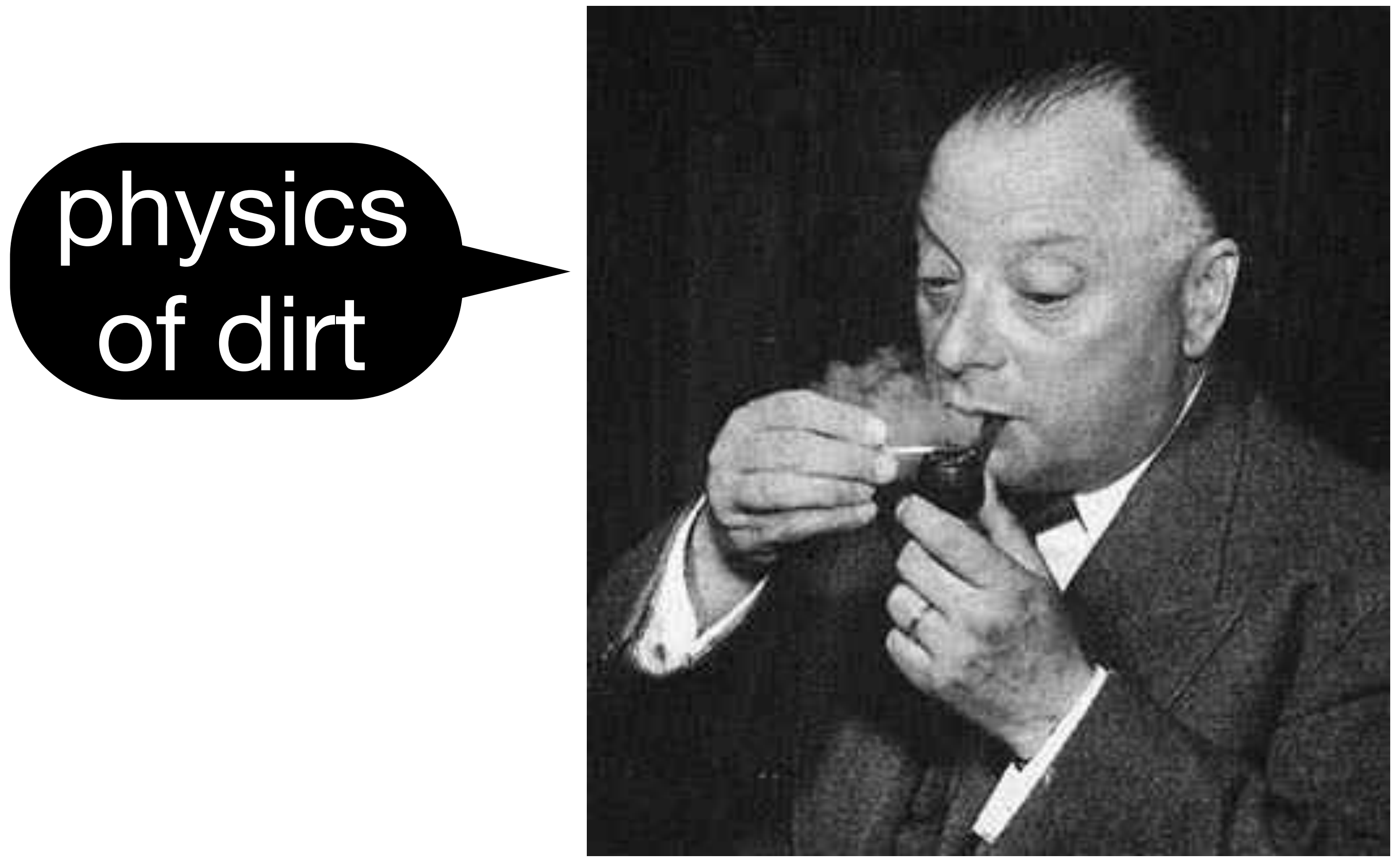
- Chiral lattice gauge theory
  - Mirror fermion (Eichten, Preskill 1986)
  - 3450 chiral fermion (Zeng et al. 2022)
- Strong CP problem in QCD
  - Alternative to Peccei-Quinn (Wang 2022)

- Interaction-reduced classification of SPT
  - 1d: $\mathbb{Z} \to \mathbb{Z}_8$ (Fidkowski, Kitaev 2009, 2010)
  - 1d, and higher dim (You et al. 2014, 2017)
- High T_c superconductors
  - LaNiO under pressure (Sun et al. 2023)
    doped SMG insulator (Lu et al. 2023)

Motivation of studying SMG







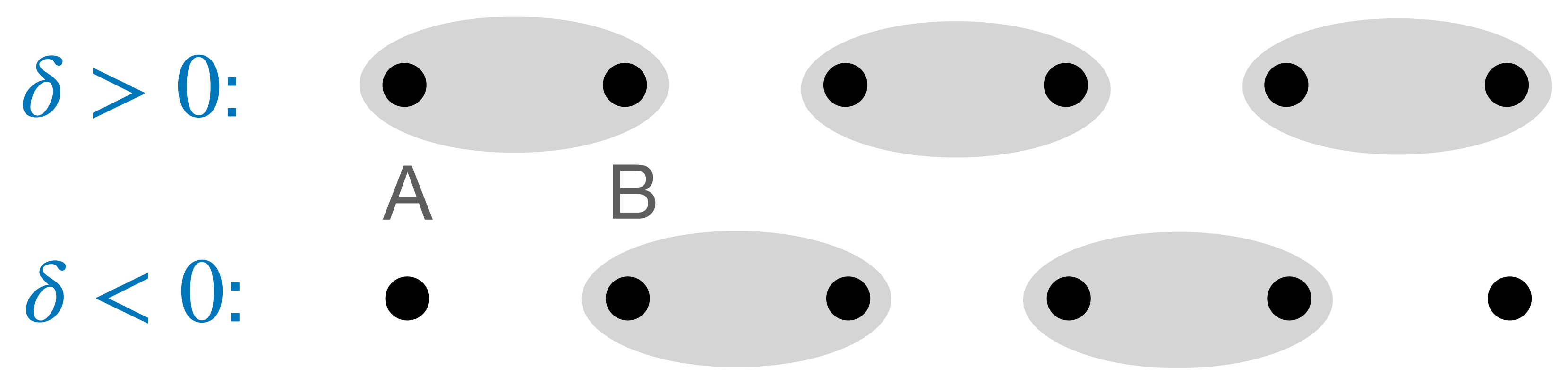
- Chiral lattice gauge theory
  - Mirror fermion (Eichten, Preskill 1986)
  - 3450 chiral fermion (Zeng et al. 2022)
- Strong CP problem in QCD
  - Alternative to Peccei-Quinn (Wang 2022)

- Interaction-reduced classification of SPT
  - 1d: $\mathbb{Z} \rightarrow \mathbb{Z}_8$ (Fidkowski, Kitaev 2009, 2010)
  - 1d, and higher dim (You et al. 2014, 2017)
- High T_c superconductors
  - LaNiO under pressure (Sun et al. 2023)

    doped SMG insulator (Lu et al. 2023)

Interaction reduced classification: Kitaev chain

- Kitaev chain with time-reversal: $T\gamma_j T^{-1} = (-1)^j \gamma_j$

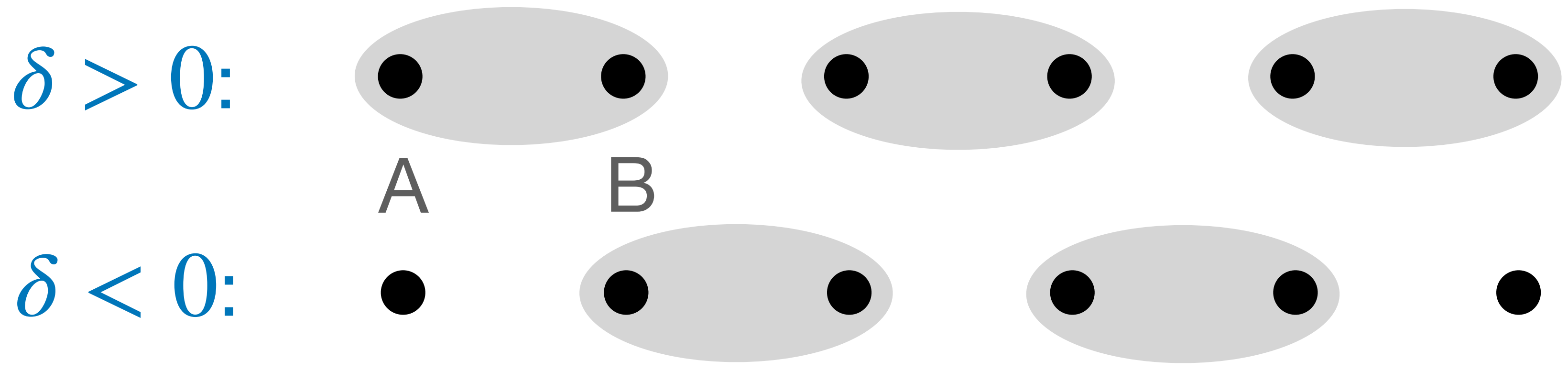


- Without interaction:
  - $Ti\gamma_j^\alpha \gamma_j^\beta T^{-1} = -i\gamma_j^\alpha \gamma_j^\beta$
  - # of edge states $\in \mathbb{Z}$

(Fidkowski, Kitaev 2010, 2011; You, Xu 2014)

Interaction reduced classification: Kitaev chain

- Kitaev chain with time-reversal: $T\gamma_j T^{-1} = (-1)^j \gamma_j$



- Without interaction:
  - $Ti\gamma_j^\alpha \gamma_j^\beta T^{-1} = -i\gamma_j^\alpha \gamma_j^\beta$
  - # of edge states $\in \mathbb{Z}$
- With interactions:
  - $T\gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma T^{-1} = \gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma$
  - # of edge states $\in \mathbb{Z}_8$
  - i.e. 8 chains = trivial

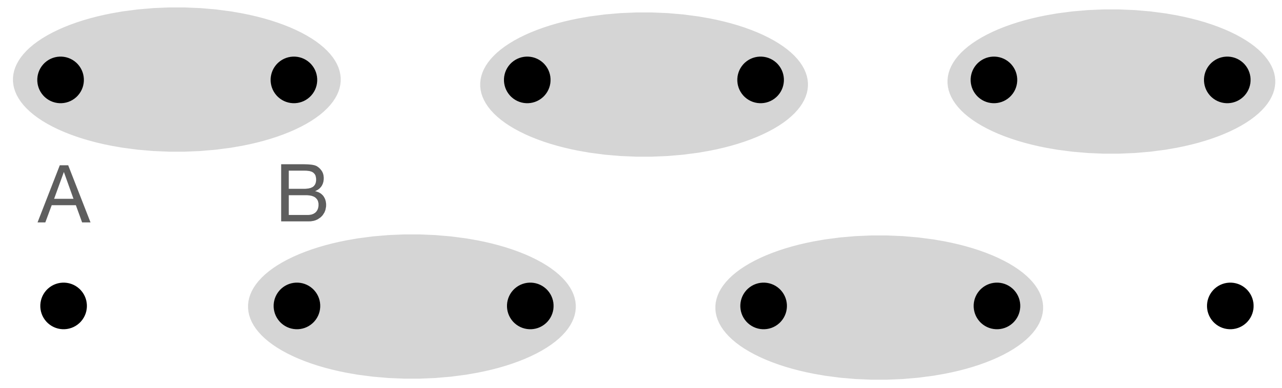
(Fidkowski, Kitaev 2010, 2011; You, Xu 2014)

Interaction reduced classification: Kitaev chain

- Kitaev chain with time-reversal: $T\gamma_j T^{-1} = (-1)^j \gamma_j$

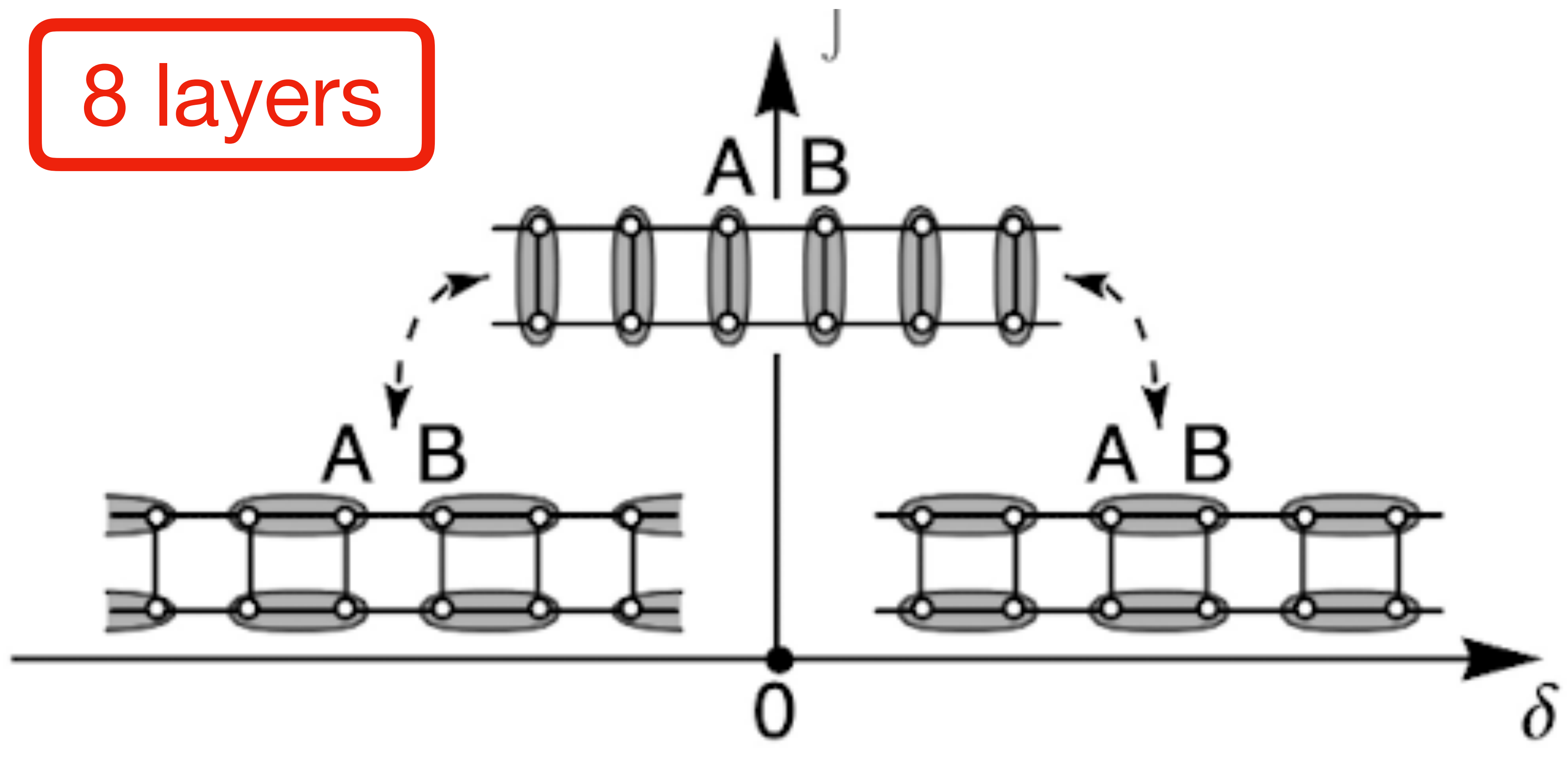
$\delta > 0$:

$\delta < 0$:



- Without interaction:
  - $Ti\gamma_j^\alpha \gamma_j^\beta T^{-1} = -i\gamma_j^\alpha \gamma_j^\beta$
  - # of edge states $\in \mathbb{Z}$
- With interactions:
  - $T\gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma T^{-1} = \gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma$
  - # of edge states $\in \mathbb{Z}_8$
  - i.e. 8 chains = trivial

8 layers

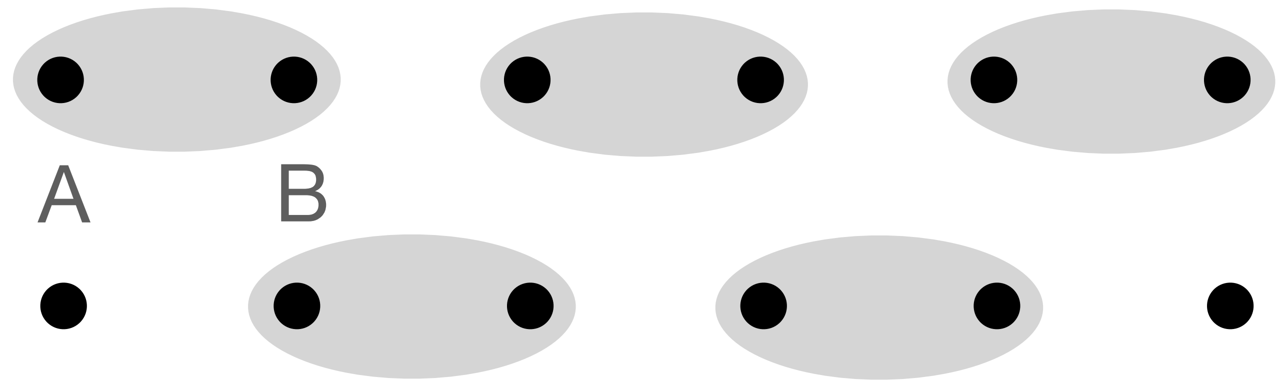


(Fidkowski, Kitaev 2010, 2011; You, Xu 2014)

Interaction reduced classification: Kitaev chain

- Kitaev chain with time-reversal: $T\gamma_j T^{-1} = (-1)^j \gamma_j$

$\delta > 0$:

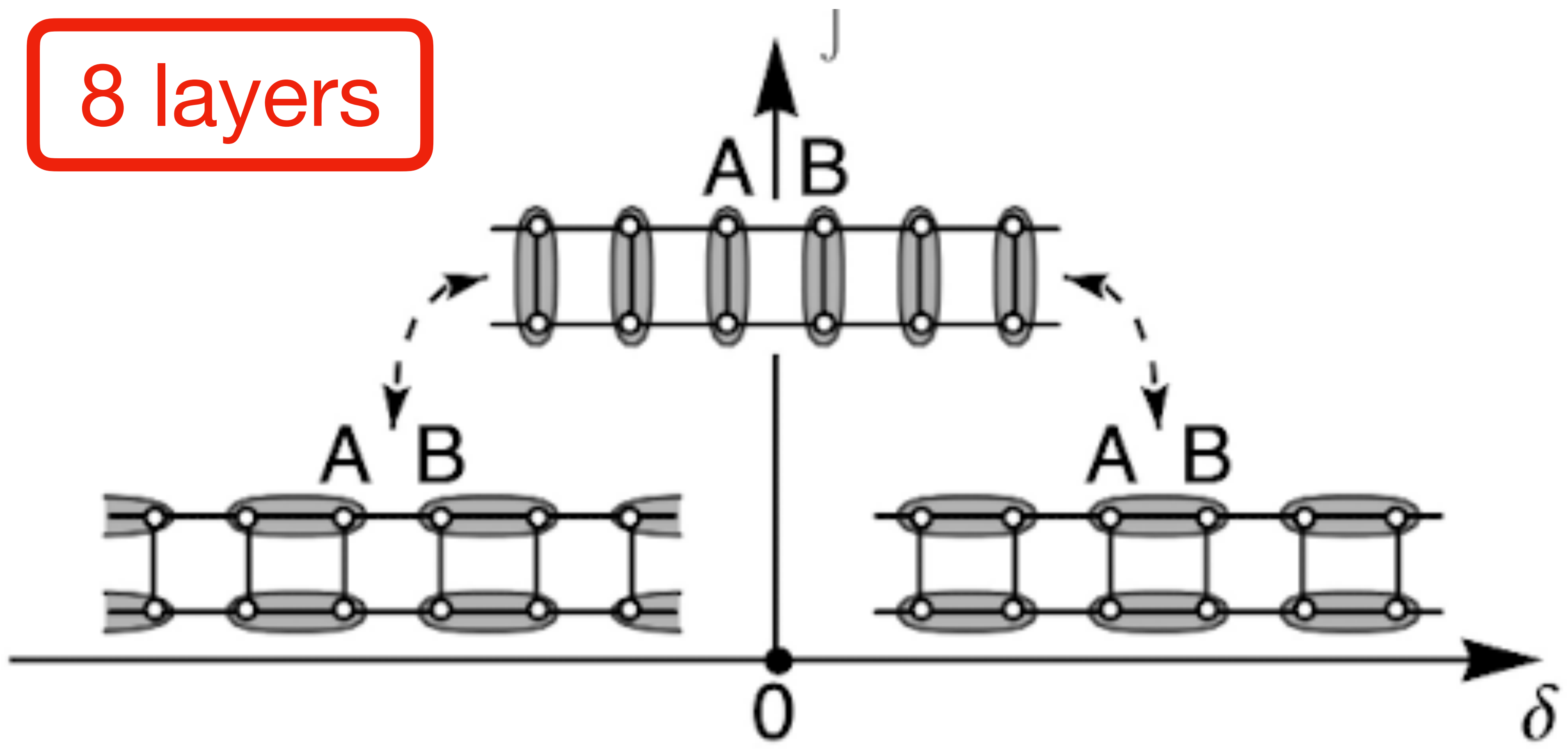


$\delta < 0$:

- Without interaction:
  - $Ti\gamma_j^\alpha \gamma_j^\beta T^{-1} = -i\gamma_j^\alpha \gamma_j^\beta$
  - # of edge states $\in \mathbb{Z}$
- With interactions:
  - $T\gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma T^{-1} = \gamma_j^\alpha \gamma_j^\beta \gamma_j^\rho \gamma_j^\sigma$
  - # of edge states $\in \mathbb{Z}_8$
  - i.e. 8 chains = trivial

(Fidkowski, Kitaev 2010, 2011; You, Xu 2014)

8 layers



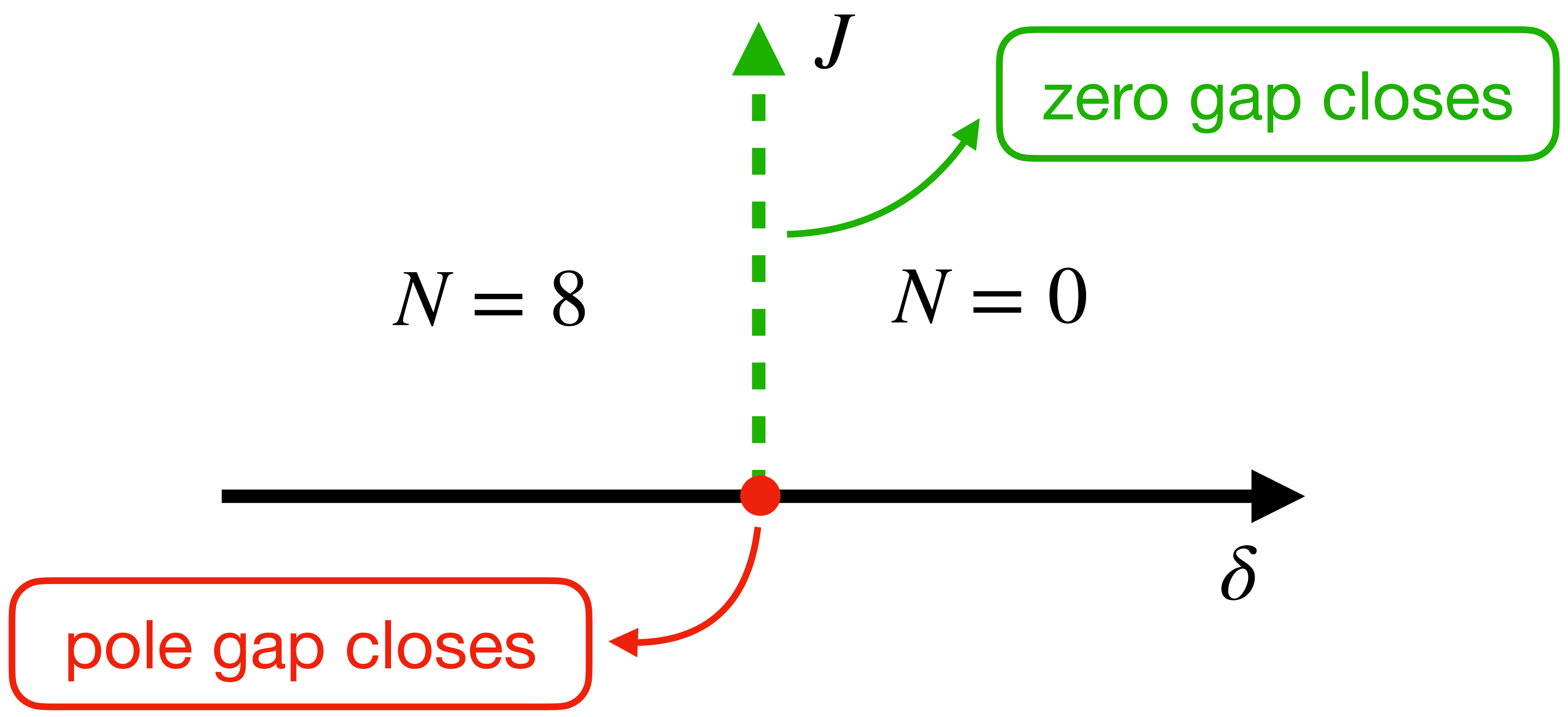
Why 8?

$\vec{S}_j^{(1)} \cdot \vec{S}_j^{(2)}$	2 spins
$f_{j,\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{j,\beta}$	2 x 2 = 4 complex fermions
$\gamma_j + i\tilde{\gamma}_j$	4 x 2 = 8 Majorana fermions

Topological indices from Green's function

- Kitaev chain

$$N = \frac{1}{4\pi i} \int dk \mathrm{Tr}[\sigma^3 G \partial_k G^{-1}]$$

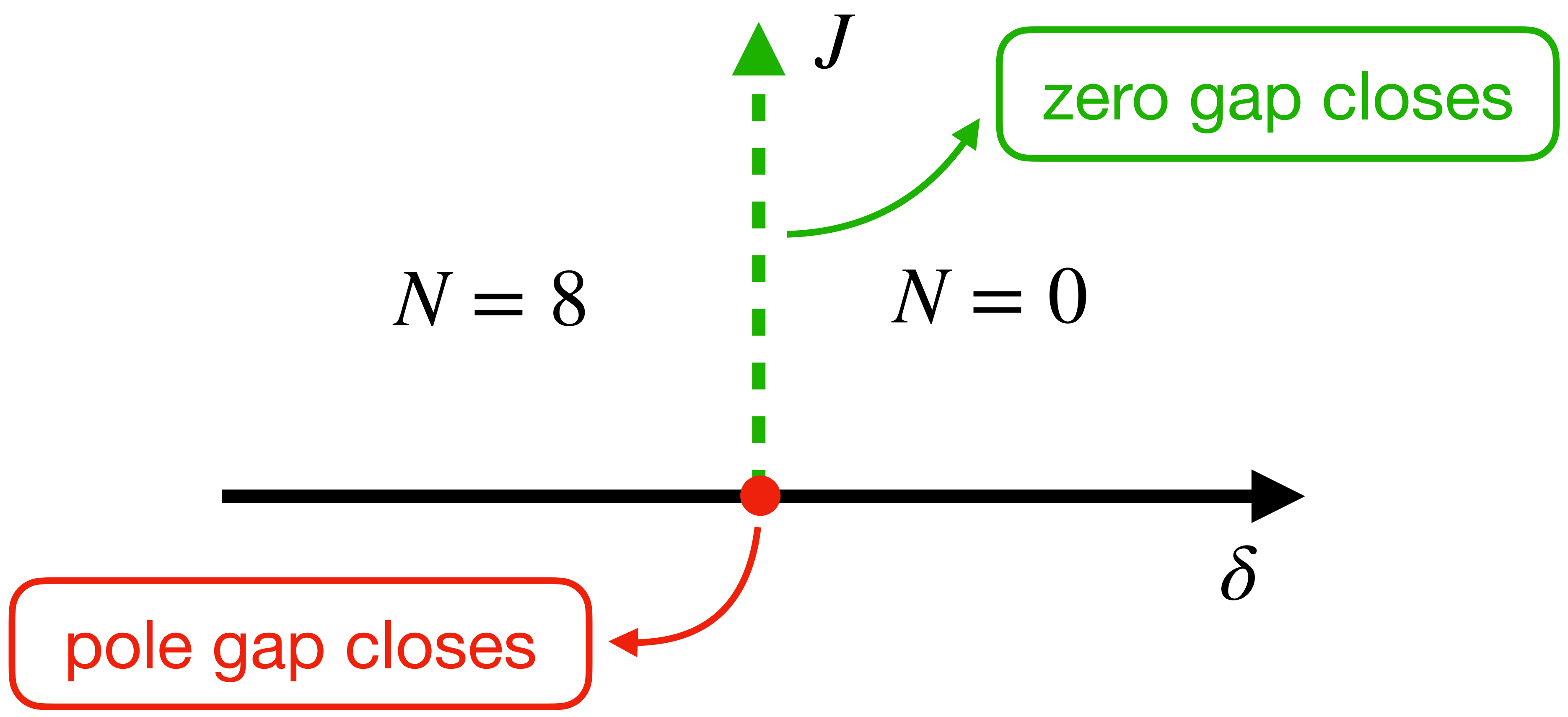


(You, Xu 2014; Zhao et al. 2023)

Topological indices from Green's function

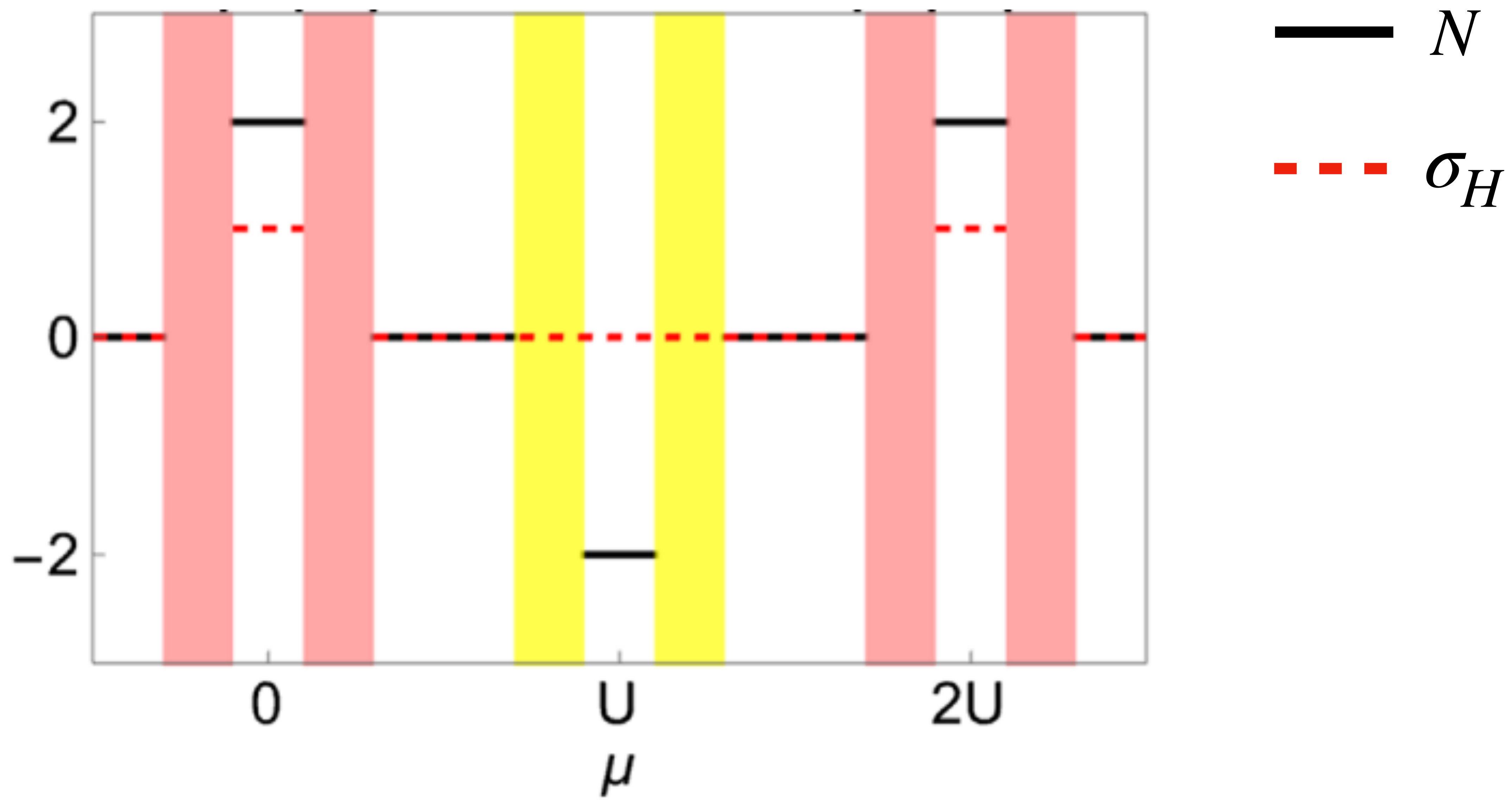
- Kitaev chain

$$N = \frac{1}{4\pi i}\int dk \mathrm{Tr}[\sigma^3 G \partial_k G^{-1}]$$



- QAH insulator (Hatsugai-Kohmoto interaction)

$$N = \frac{\epsilon_{\alpha\beta\gamma}}{24\pi^2}\int d\omega dk \mathrm{Tr}[G^{-1}\partial_{k_\alpha} G G^{-1}\partial_{k_\beta} G G^{-1}\partial_{k_\gamma} G]$$



(You, Xu 2014; Zhao et al. 2023)

Recap

Previously

- Interacting Kitaev chain: $\mathbb{Z} \to \mathbb{Z}_8$.
  - Critical point gapped without SSB.
  - Emergence of Green's function zeros.

Next

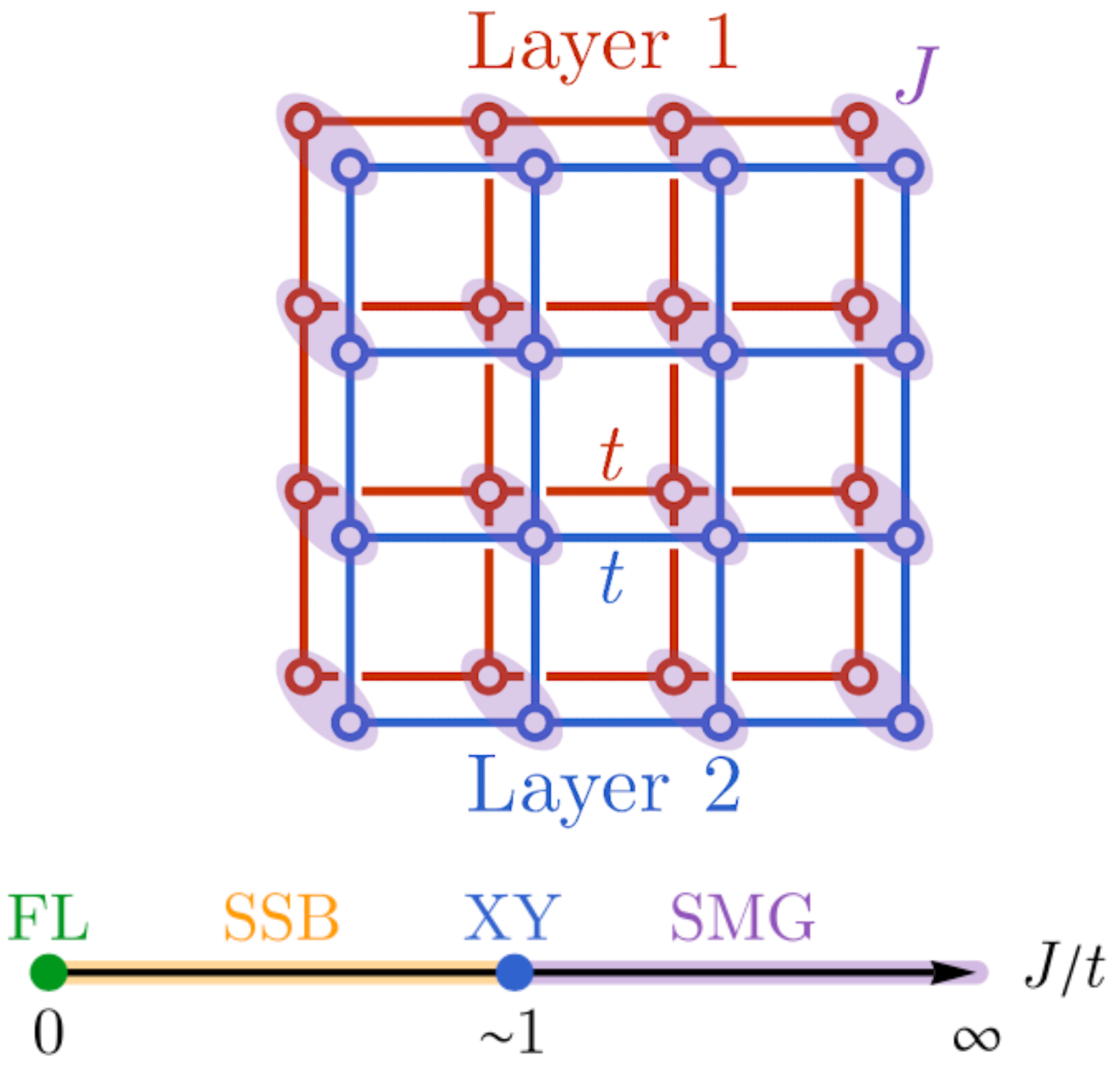
- Generalize to critical phases.
  - FS

Zeros in Fermi surface SMG

- Model: $H = -t \sum_{\langle ij \rangle, l, \sigma} (c^\dagger_{il\sigma} c_{jl\sigma} + \text{H.c.}) + J \sum_i \boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2}$ with $S^\alpha_{il} = \frac{1}{2} c^\dagger_{il} \sigma^\alpha c_{il}$ and $\nu_l = 1/2$.

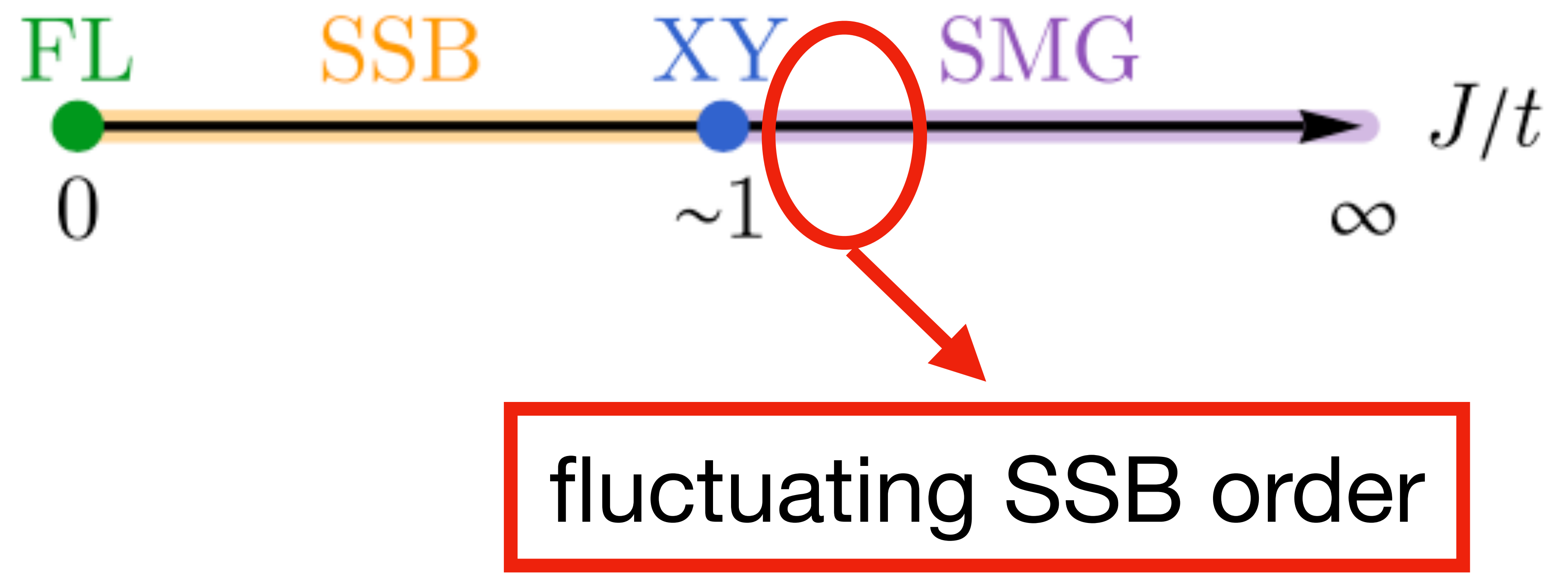
- Coupling strengths:
  - $J/t \lesssim 1$: SSB due to FS nesting
  - $J/t \gtrsim 1$: weak-coupling SMG (next slide)
  - $J/t \gg 1$: strong-coupling SMG

$$|GS\rangle = \otimes_i \frac{1}{\sqrt{2}} \left(| \uparrow_{i1} \downarrow_{i2} \rangle - | \downarrow_{i1} \uparrow_{i2} \rangle \right)$$



(Lu, Zeng, You 2023)

Weak coupling SMG: from disordering SSB



SSB:

$$G_{\mathrm{SSB}}(\omega, \boldsymbol{k}) = \frac{\omega\sigma^0 + \epsilon_{\boldsymbol{k}}\sigma^3 + \mathrm{Re}\,\Delta\sigma^1 + \mathrm{Im}\,\Delta\sigma^2}{\omega^2 - \epsilon_{\boldsymbol{k}}^2 - |\Delta|^2}$$

$\Sigma(\omega, \boldsymbol{k}) =$  $= \quad \hat{\Delta}^\dagger G_0(\omega, \boldsymbol{k})\hat{\Delta} = \dfrac{\Delta_0^2}{\omega\sigma^0 + \epsilon_{\boldsymbol{k}}\sigma^3}$

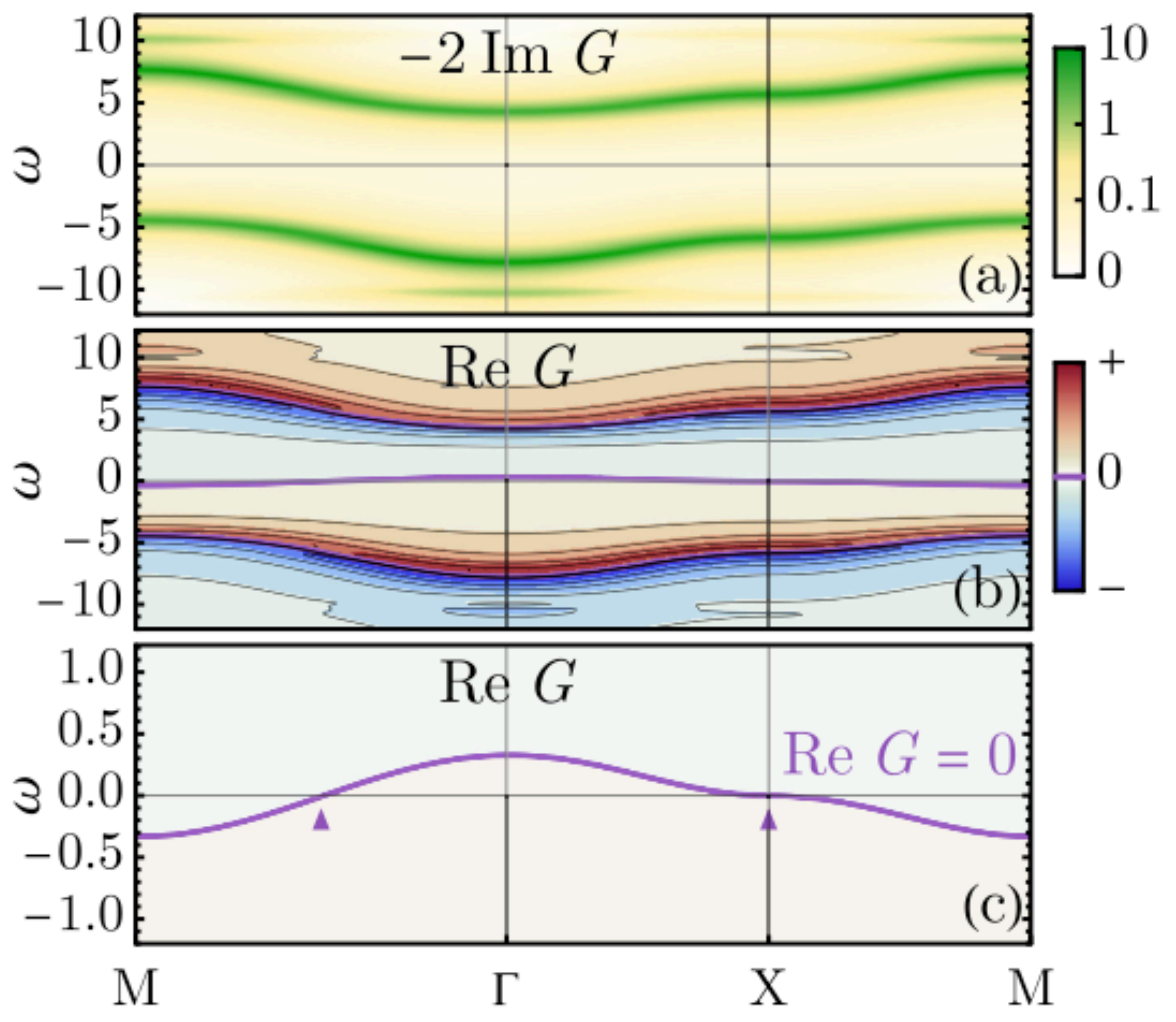
$$G(\omega, \boldsymbol{k}) = (G_0(\omega, \boldsymbol{k})^{-1} - \Sigma(\omega, \boldsymbol{k}))^{-1}$$
$$= \frac{\omega\sigma^0 + \epsilon_{\boldsymbol{k}}\sigma^3}{\omega^2 - \epsilon_{\boldsymbol{k}}^2 - \Delta_0^2}.$$

Strong coupling SMG (next slide):

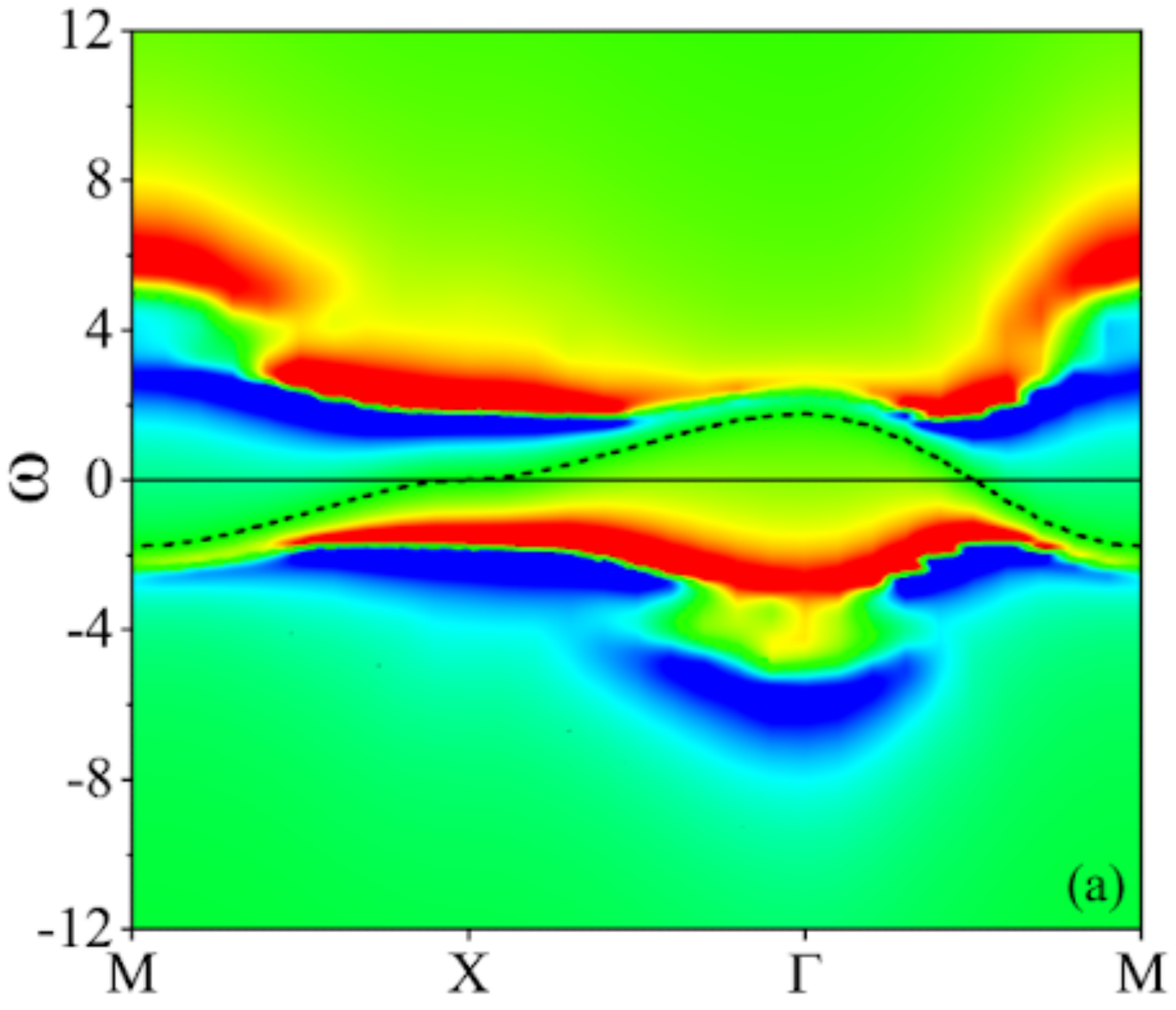
$$G_{\mathrm{SMG}}(\omega, \boldsymbol{k}) = \frac{\omega + \alpha\epsilon_{\boldsymbol{k}}/J^2}{(\omega - \epsilon_{\boldsymbol{k}}/2)^2 - J^2}$$

Strong coupling SMG: numerics

- **Cluster perturbation**
  (Lu, Zeng, You 2023)



- **Quantum Monte Carlo**
  (Chang, Guo, You, Li 2023)

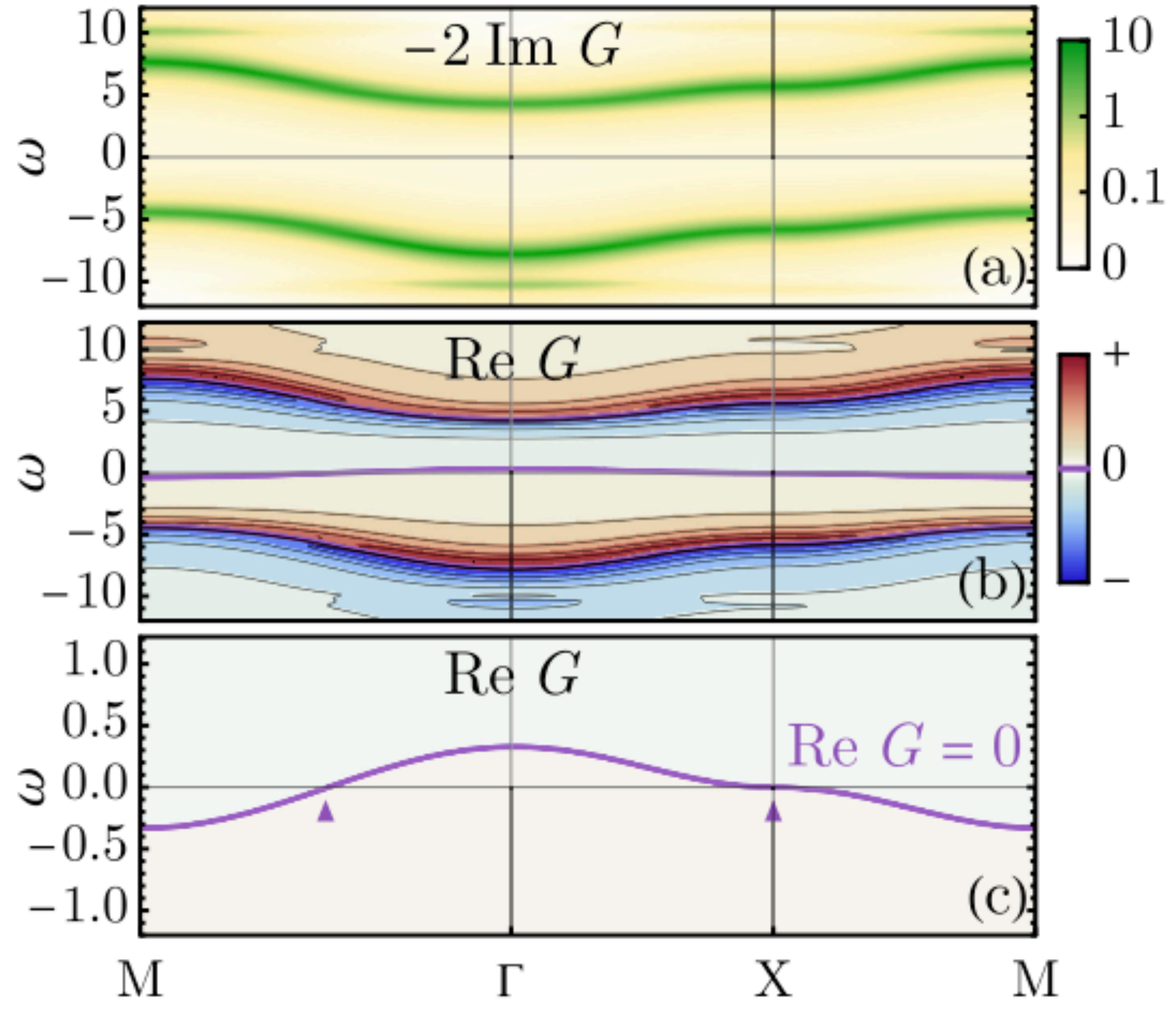


$$G_{\mathrm{SMG}}(\omega, \boldsymbol{k}) = \frac{\omega + \alpha\epsilon_{\boldsymbol{k}}/J^2}{(\omega - \epsilon_{\boldsymbol{k}}/2)^2 - J^2}$$

Luttinger theorem in gapped phase!

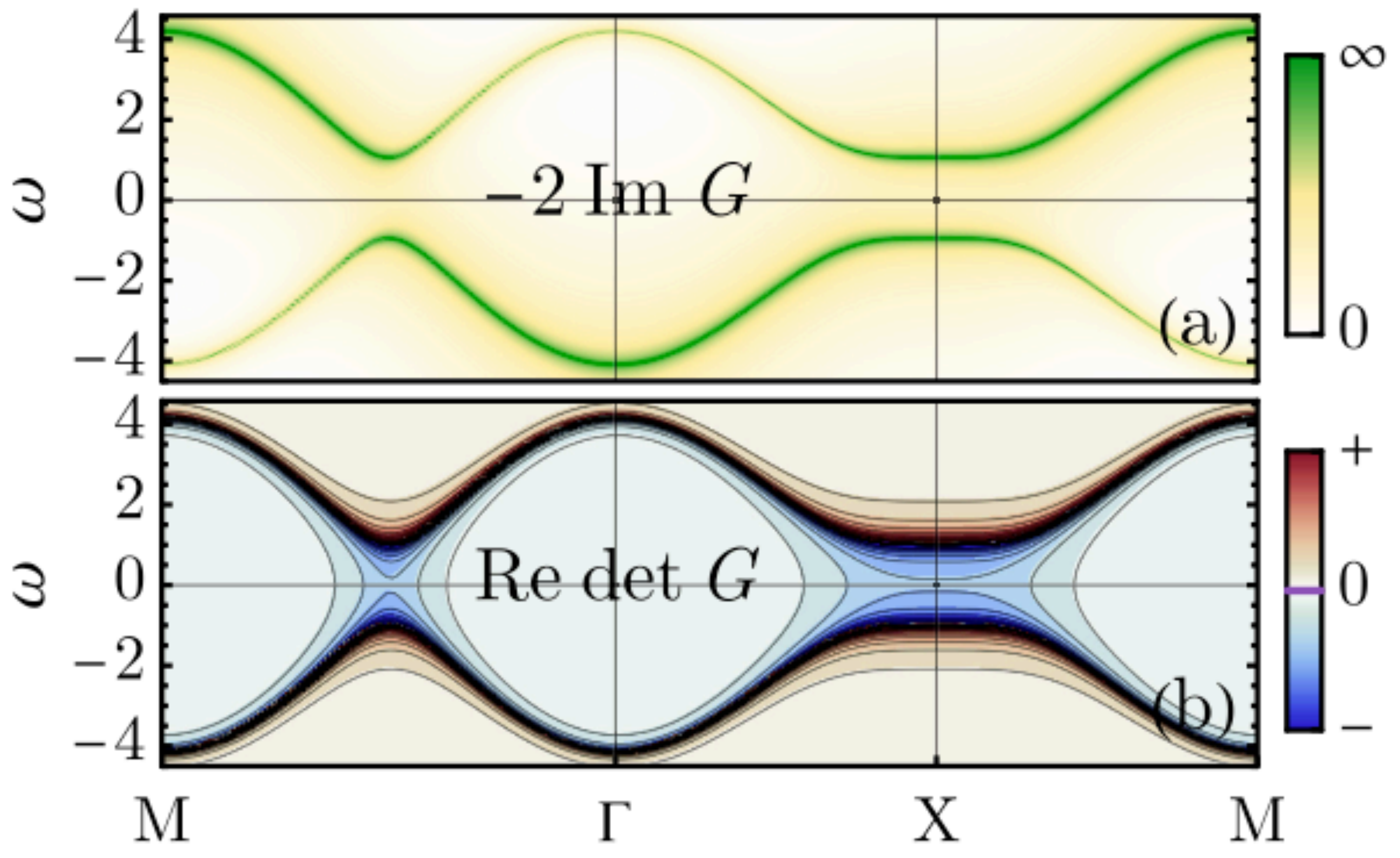
Green's function zeros: SMG v.s. SSB

SMG

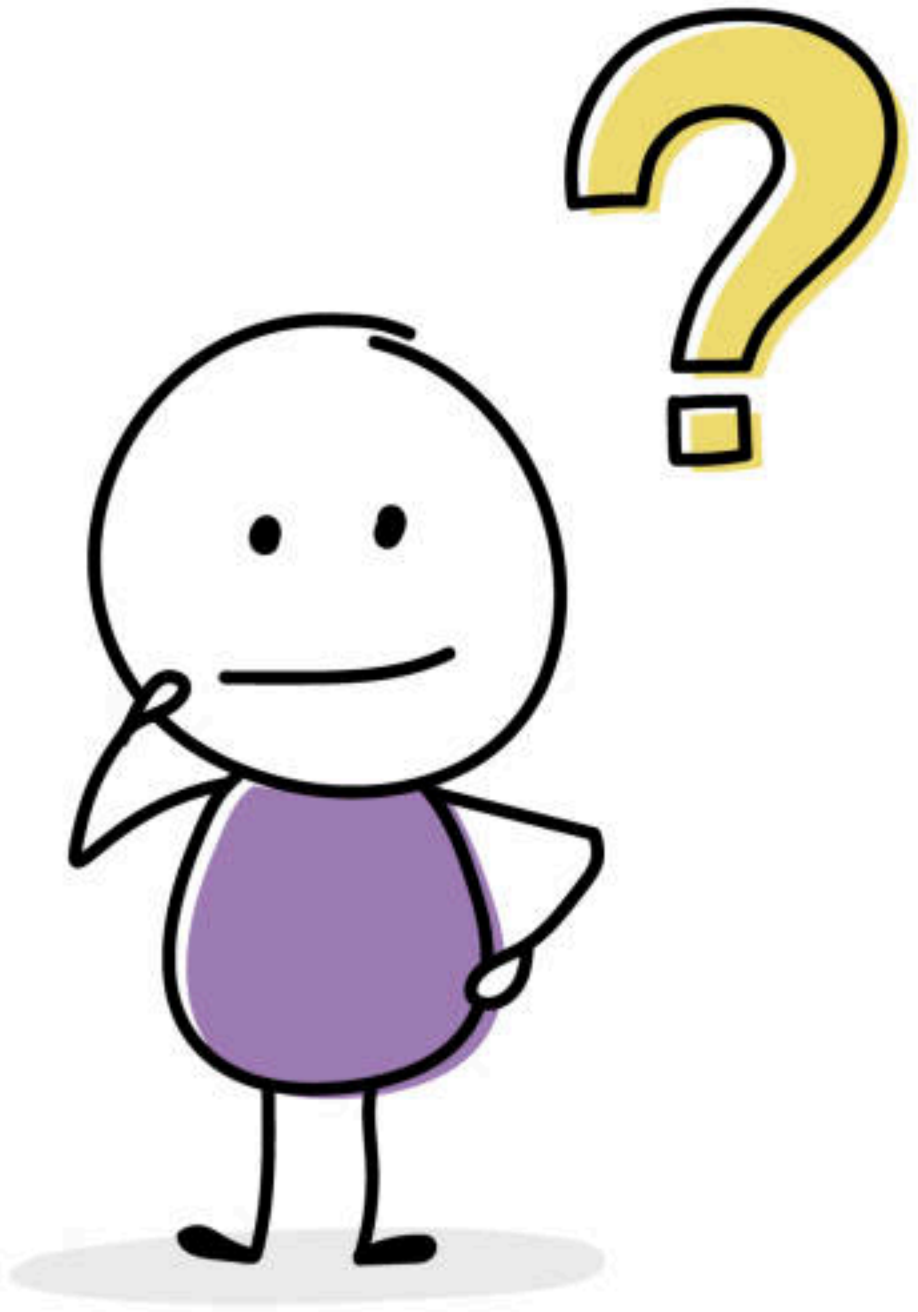


SSB

$$G_{\mathrm{SSB}}(\omega, \boldsymbol{k}) = \frac{\omega\sigma^0 + \epsilon_k\sigma^3 + \mathrm{Re}\,\Delta\sigma^1 + \mathrm{Im}\,\Delta\sigma^2}{\omega^2 - \epsilon_{\boldsymbol{k}}^2 - |\Delta|^2}$$



(Lu, Zeng, You 2023)



Q: OK, there is zero, so what?

Zero-pole duality and EM response

- EFT from Green's function

$$S[\psi, A] = \int dx\ \bar{\psi} G^{-1}(i\partial - A)\psi \quad \to \quad S[A] = \mathrm{Tr}\log G(i\partial - A)$$

- Zero-pole duality

$$G_{\mathrm{Dirac}}(k) \sim \frac{1}{\gamma^\mu k_\mu}, \quad G_{\mathrm{SMG}}(k) \sim -\gamma^\mu k_\mu, \qquad G(k) \to G(-k)^{-1}$$

- Current correlations under the duality

$$S[A] \to -S[-A]$$

$$\Pi_n^{\mu_1 \ldots \mu_n} \to (-1)^{n+1}\ \Pi_n^{\mu_1 \ldots \mu_n} \quad \text{with} \quad \Pi_n^{\mu_1 \ldots \mu_n} \equiv \delta_{A_{\mu_1}} \ldots \delta_{A_{\mu_n}} S[A]$$

- Paradox for EM response

$$\Pi^{\mu\nu}_{2,\mathrm{SMG}} = -\Pi^{\mu\nu}_{2,\mathrm{Dirac}}, \quad \Pi^{\mu\nu\rho}_{3,\mathrm{SMG}} = \Pi^{\mu\nu\rho}_{3,\mathrm{Dirac}}, \ \ldots$$

(You, He, Xu, Vishwannath 2017; Golterman, Shamir 2023; Zeng, Xu, Lu, You 2024)

Current operator

- Naive definition of current from EFT with minimal coupling

$$J_\mu \sim \int dk\ \bar{\psi}_k \frac{\delta G^{-1}}{\delta A_\mu} \psi_k$$

- Non-interacting:

$$G = \frac{1}{\omega - h_k} \Rightarrow J_\mu \sim \sum_k \bar{\psi}_k \frac{\partial h_k}{\partial k} \psi_k$$

- Interacting: Zero leads to divergent current

$$G \to 0 \Rightarrow G^{-1} \to \infty \Rightarrow J_\mu \to \infty$$

- Proper definition of current operator

$J_\mu = J_\mu^{\text{free}} + J_\mu^{\text{int}}, \quad J_\mu^{\text{int}} = 0$ for on-site interactions

(Zeng, Xu, Lu, You 2024)

Optical conductivity: ideal SMG limit

- Concrete lattice model

$$H = -\sum_{ij} t_{ij} e^{iA_{ij}} c_i^\dagger c_j - g \sum_i c_{i1}^\dagger c_{i2}^\dagger c_{i3} c_{i4} + \mathrm{h.c.}$$

- Ideal SMG limit: $g \to \infty$

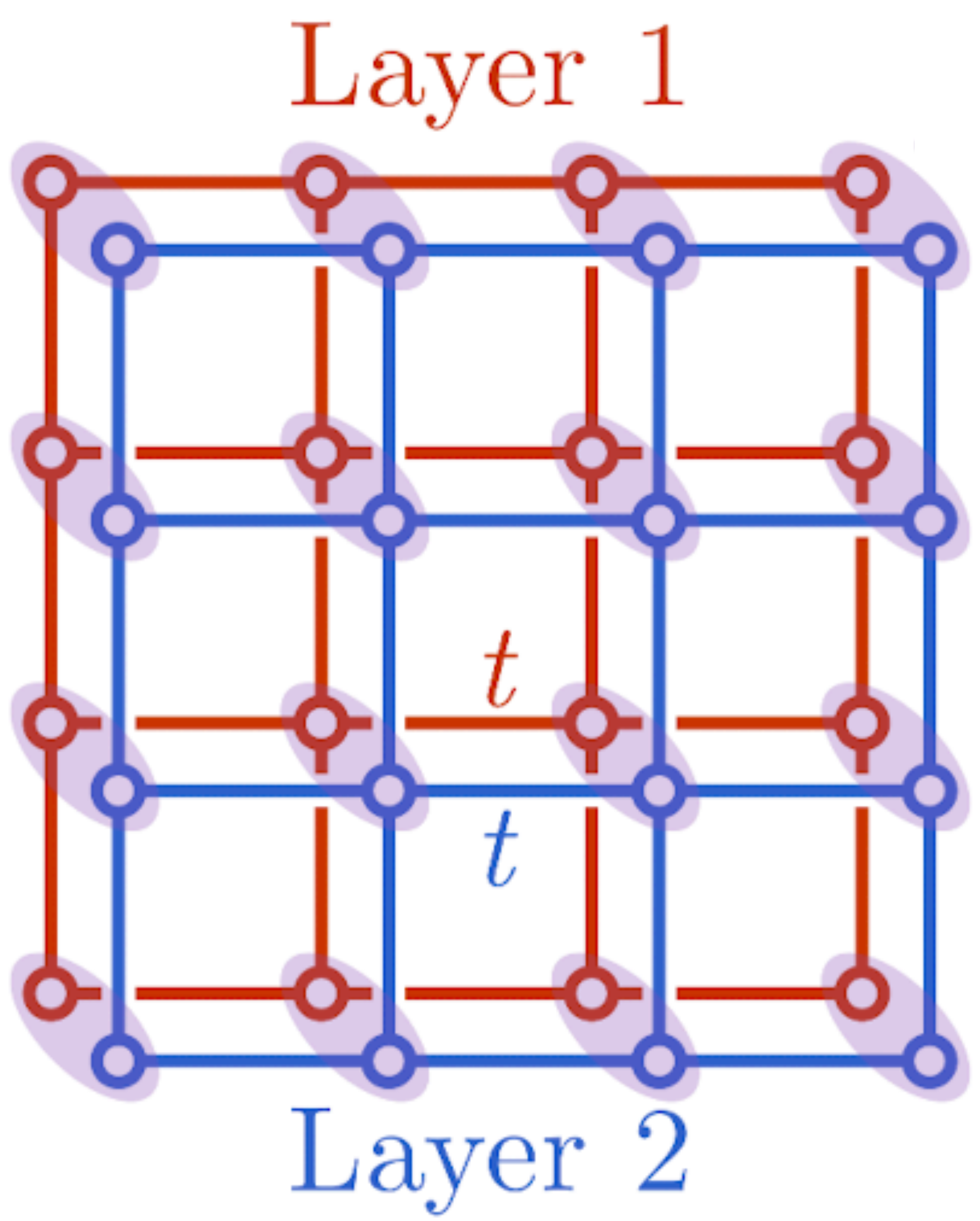
$$|\Psi_{\mathrm{SMG}}\rangle \sim \prod_i (c_{1i}^\dagger c_{i2}^\dagger - c_{3i}^\dagger c_{i4}^\dagger)|0\rangle$$

- Current and correlations

$$J_{ij} = \frac{\delta H}{\delta A_{ij}} = -\mathrm{i} t_{ij} c_i^\dagger c_j + \mathrm{h.c.}$$

$$\Pi(t) = -i\langle [J_{ij}(t), J_{kl}(0)]\rangle \theta(t)$$

$$\Rightarrow \mathrm{Re}\sigma(\omega) = -\frac{1}{\omega}\mathrm{Im}\Pi(\omega) \sim \delta(\omega - 2g) + \delta(\omega + 2g)$$



(Zeng, Xu, Lu, You 2024)

Optical conductivity: beyond ideal limit

$$G_{\mathrm{SMG}}(\mathbf{k},\omega) = \frac{\omega\sigma^0 + \epsilon_{\mathbf{k}}\sigma^1}{\omega^2 - \epsilon_{\mathbf{k}}^2 - \Delta^2}$$

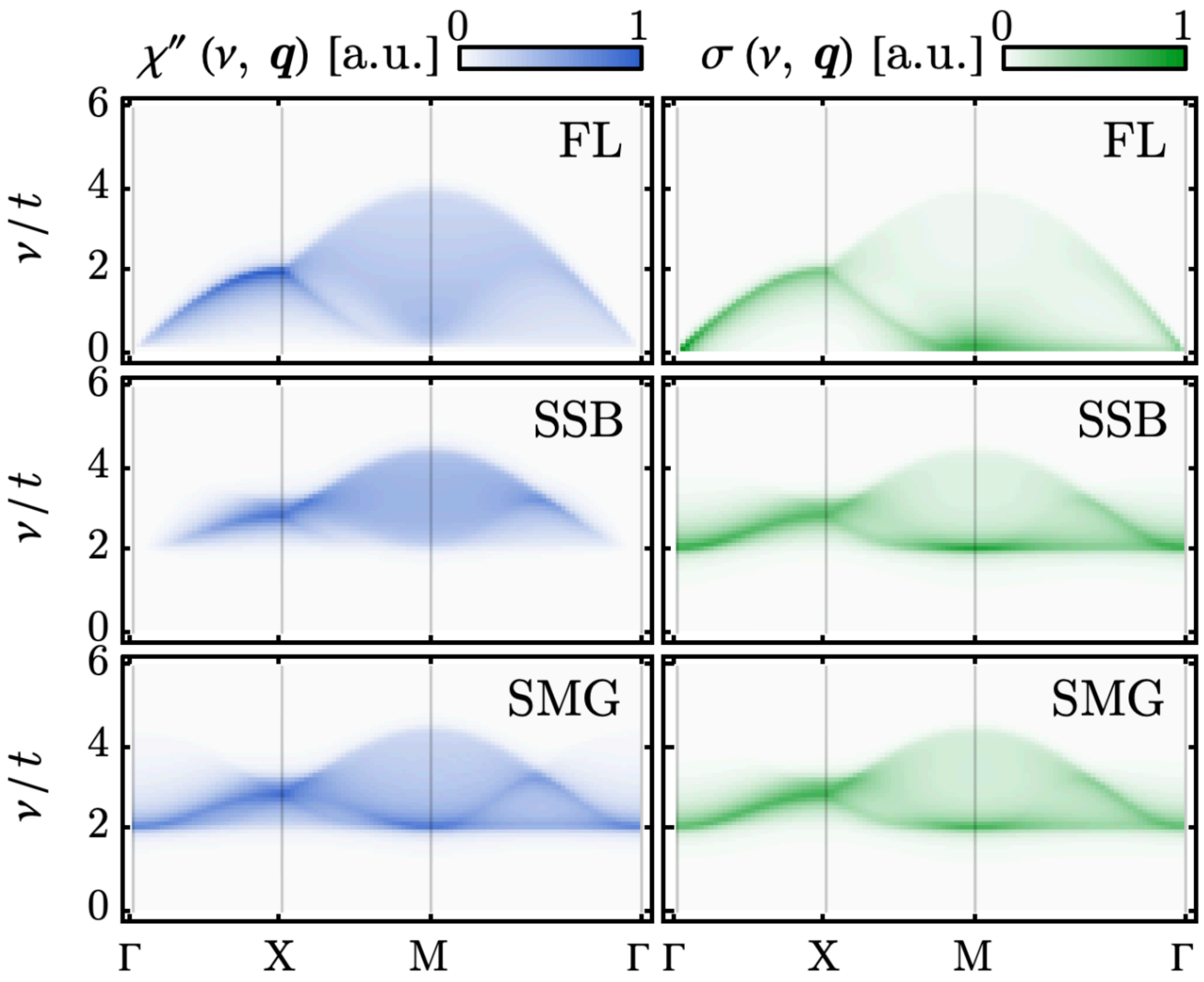
$$\Pi^{00}(\mathbf{q},\nu) = \mathrm{tr}[G(\mathbf{k},\omega)G(\mathbf{k}+\mathbf{q},\omega+\nu)]$$

$$\Pi^{ij}(\mathbf{q},\nu) = \mathrm{tr}[G(\mathbf{k},\omega)v_i G(\mathbf{k}+\mathbf{q},\omega+\nu)v_j]$$

⬇

$$\chi''(\mathbf{q},\nu) = -2\mathrm{Im}\Pi^{00}(\mathbf{q},\nu+i0^+)$$

$$\sigma(\mathbf{q},\nu) = -\frac{1}{\nu}\mathrm{Im}\Pi^{ii}(\mathbf{q},\nu+i0^+)$$



(Zeng, Xu, Lu, You 2024)

Can zeros become physical?

- SMG insulator with onsite interaction: No
- SMG insulator with non-onsite interaction (e.g. 3450 model): Not so clear
- Recent: zeros in other gapped phases
  - **Bulk** zeros in Mott phase (H_{eff} for zeros)
    - Symmetry-constrained zeros (Setty et al 2023)
    - Berry curvature and flux quantization (Chen et al 2024)
    - Luttinger count and Hall conductivity (Setty et al 2023)
  - **Boundary** zeros in topological Mott insulators (Wagner et al 2023)
- Perspective from anomalies (Su et al 2024)

Strong coupling: Cluster perturbation

Lehmann:

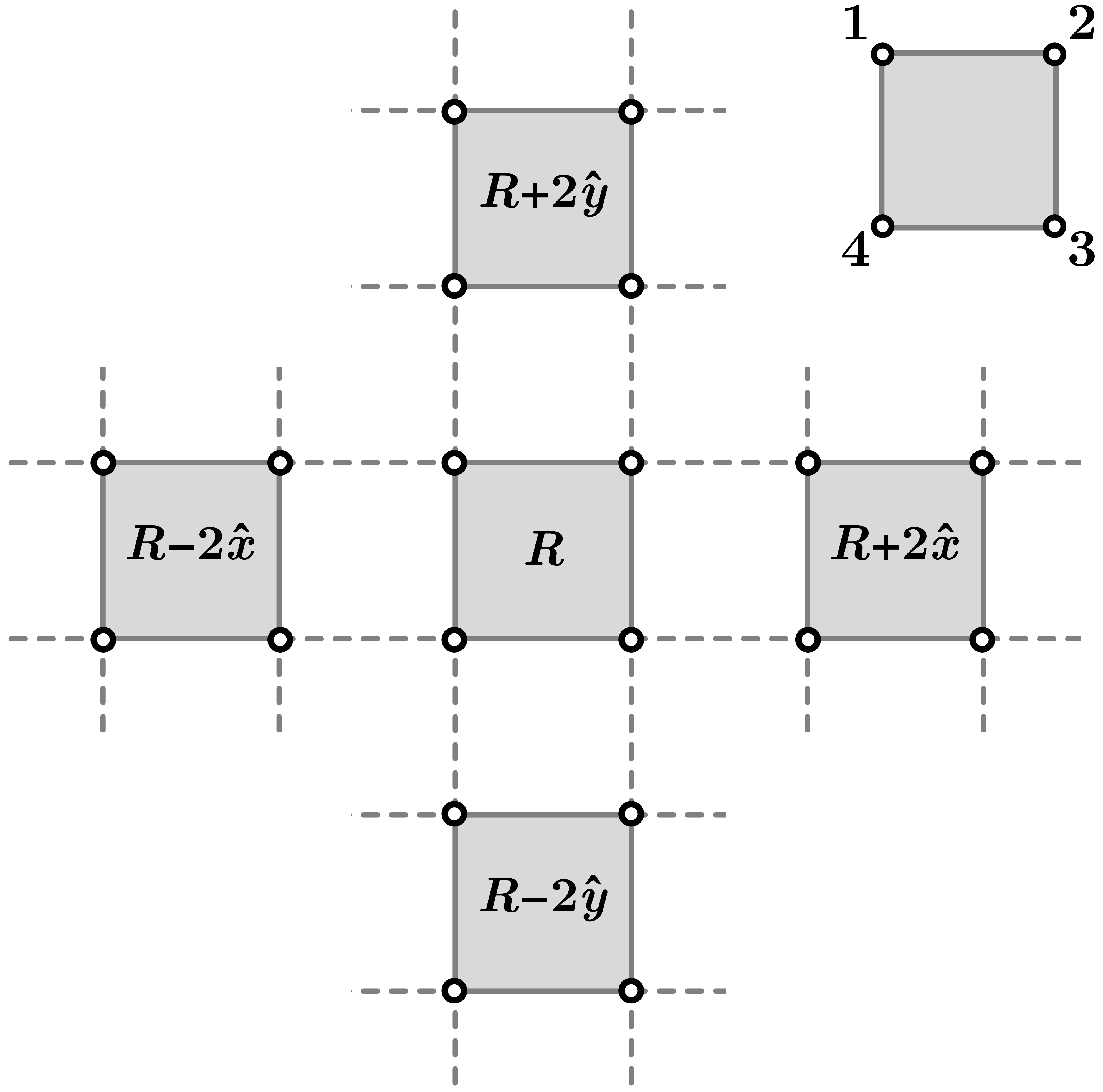
$$G_0(\omega)_{ij} = \sum_{m>0} \frac{\langle 0|c_i|m\rangle\langle m|c_j^\dagger|0\rangle}{\omega - (E_m - E_0)} + \frac{\langle m|c_i|0\rangle\langle 0|c_j^\dagger|m\rangle}{\omega + (E_m - E_0)}$$

RPA:

$$G(\omega, \boldsymbol{k})_{ij} = \left(\frac{G_0(\omega)}{1 - T(\boldsymbol{k})G_0(\omega)}\right)_{ij}$$

Unfold:

$$G(\omega, \boldsymbol{k}) = \frac{1}{L}\sum_{i,j} e^{-i\boldsymbol{k}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)} G(\omega, \boldsymbol{k})_{ij}$$



Experimental probe with ARPES

ARPES $\Rightarrow$ $A(\omega, k) \sim \mathrm{Im}G$ $\Rightarrow$ $\mathrm{Re}G$